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The shapes of dark matter dominated dwarf spheroidal stellar density profiles

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ABSTRACT

A Bayesian likelihood analysis of the stellar density profile shapes of several dark matter dominated Galactic dwarf spheroidal galaxies (dSphs) has been completed. This analysis indicates that the inner power law, $\rho \propto r^{-\gamma}$, of each of these profiles satisfies $\gamma < 1.21$ at greater than 99% confidence with a slight preference for $\gamma \sim 0.7$. Speculating that this bound on the stellar profiles may be related to the inner power law of the underlying dark matter distribution reveals a statistical discrepancy with the values favoured by recent N-body simulations of cold dark matter (CDM), $\gamma_{CDM} \sim 1.4 - 1.5$.

Key words: methods statistical – galaxies: dwarf – galaxies: haloes – galaxies: structure – dark matter.

1 INTRODUCTION

The nature of the dark matter remains one of the prime areas of astrophysical and cosmological research. There has been some controversy recently concerning cold collisionless dark matter (CDM), arguably the simplest solution to dark matter problem. Assuming that satellite dark haloes can be associated with dwarf spheroidal galaxies (dSphs), CDM appears to overpredict by two orders of magnitude the number of these satellite haloes surrounding the Galactic halo (Moore et al. 1999a). Recent N-body simulations of CDM also indicate that dark matter haloes have cuspy cores on subgalactic scales, with an inner exponent $\gamma_{CDM} \sim 1.4 - 1.5$ (Fukushige & Makino 1997; Moore et al. 1999b; Jing & Suto 2000; Subramanian et al. 2000; Bullock et al. 2001), up from the value found in earlier simulations of $\gamma_{CDM} \sim 1.0$ (Navarro, Frenk, & White 1996, hereafter NFW).

Recent detailed, high resolution observations of the rotational motion of LSB galaxies indicate their core density profiles are substantially flatter than CDM predictions (de Blok et al. 2001; see also Borriello & Salucci 2001). (For earlier discussion and debate, see for example van den Bosch & Swaters (2000) and Moore et al. (1999b)).

Since the discrepancy between CDM theory and observation is greatest on small scales, it is natural to seek to sharpen it further by examining the smallest, nearest galaxies, namely dwarf spheroidal galaxies. Galactic dSphs have long been recognised as excellent probes of the nature of the dark matter. Besides being the smallest known galaxies they also possess the highest known M/L ratios. Observationally, their proximity means that individual stars can be resolved, so that beam smearing effects of the type discussed by van den Bosch & Swaters (2000) for LSBs are not an issue. Unfortunately, whilst velocity measurements have established the presence of large amounts of dark matter, they have hitherto not been sufficiently accurate to determine the radial profile of the dark matter density (Armandroff, Pryor & Olszewski 1997).

However, the stellar density profile is rather well established (Irwin & Hatzidimitriou 1995) and enough data exists for a detailed comparison with the theoretically predicted dark matter profiles. This comparison is only meaningful if the stars trace the mass, a very strong assumption, which is hard to justify. Nevertheless, it seems a reasonable first step to fit dSph stellar profiles to the parametric form used in theoretical studies of dark matter profiles, whilst acknowledging that any conclusion on the dSph dark matter profile must await more accurate velocity measurements. As both theory and observations develop, we anticipate that a more complete likelihood analysis, of the type performed here but including both density and velocity data, should become possible.

One should also point out that at least for some star formation histories, comparing the stellar and dark matter profiles is not without meaning. If the bulk of the stars formed before the dSph dark matter haloes, they would thereafter cluster under gravity just like dark matter particles. If, on

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the other hand, stars formed after the dark matter haloes, from baryons which were heated when the Universe was reionised at $z \gtrsim 5$, or alternatively if the stars in dSphs were accreted from other parts of the galaxy, there would not necessarily be any simple relation between the stellar, and dark matter density profiles.

CDM theory over-predicts both the number density, and the velocity dispersion of haloes with the masses of dwarf spheroidals (Bode, Ostriker, & Turok 2000). If CDM is to be viable, there must exist some mechanism (such as stellar feedback) to suppress star formation in most, but not all, low mass haloes. The surviving low mass galaxies would then be those in which stars formed earliest, and those which possessed the highest central density. These selection effects would tend to favor haloes with steeper-than-typical inner profiles and higher central densities. Comparing a typical CDM halo profile against observed dark matter profiles would actually be conservative in this respect.

In a similar way we may also speculate on how compatible other theories of dark matter are with observations of dark matter dominated dwarf spheroidals. For example, it has been suggested that a warm dark matter (WDM) scenario may better match the observed number of satellite galaxies predicted for the local group (Avila-Reese et al. 2001) and so a mechanism to suppress star formation may be less necessary. Additionally, stars may be a more viable tracer of the dark matter in WDM theories as the recollapse of small WDM haloes is expected to occur considerably later than in the CDM case. However, despite these advantages, theories of WDM may predict haloes cuspy enough to be cause for concern as in the CDM case (Eke et al. 2001).

2 METHODOLOGY

We have studied the radial distribution of stellar matter in the dark matter dominated Galactic dSphs using the framework of Bayesian likelihood analysis (Gregory & Loredo 1992).

The stellar density profile was assumed to take the smoothly interpolating double power law form of equation (1) (e.g., Kravtsov et al. 1998).

$$\rho(r) = \frac{\rho_0}{(r/r_0)^\gamma [0.5 + 0.5(r/r_0)^\alpha]^{(\beta-\gamma)/\alpha}} \tag{1}$$

Using this form yields $\rho \propto r^{-\gamma}$ for radial distances smaller than r_0, and $\rho \propto r^{-\beta}$ for radial distances larger than r_0. The parameter α controls the scale over which the power law shifts from γ to β. Roughly speaking, this change occurs between radii $e^{-1/\alpha} r_0$ and $e^{1/\alpha} r_0$. This density profile accommodates a wide range of useful models, including a modified King-like profile (King 1962; Rood et al. 1972) for $(\alpha, \beta, \gamma) = (2, 3, 0)$, the NFW profile for $(\alpha, \beta, \gamma) = (1, 3, 1)$, and the profile suggested by recent N-body CDM simulations $(\alpha, \beta, \gamma) = (1, 3, 1.5)$.

Projecting onto the celestial sphere gives the effective two dimensional stellar surface density.

$$\Sigma(r_\perp) = 2 \int_0^\infty \rho\left(\sqrt{r_\perp^2 + z^2}\right) dz \tag{2}$$

Currently, the best observational determination of stellar density as a function of radius in the dSphs of interest is the work of Irwin & Hatzidimitriou (1995), and can be summarised in this analysis by a vector $\widetilde{\Sigma}_{obs}$ which contains the mean density at a discrete set of radii and an associated covariance matrix $\widetilde{C}_{obs}$.

For each model $(\alpha, \beta, \gamma, r_0, \rho_0)$ we have calculated the conditional relative likelihood $\mathcal{L}(\alpha, \beta, \gamma, r_0, \rho_0 | \widetilde{\Sigma}_{obs})$ – a χ^2 statistic in practice – to quantify the compatibility of a particular model with observations.

The chief advantage of the Bayesian formalism is that it allows one to assess the relative likelihood of a particular subclass of the models by integrating $\mathcal{L}$ over the relevant equiclassification surfaces in the model space. To assess the relative likelihood of different values of γ we integrate $\mathcal{L}$ over all models with the same γ but different $(\alpha, \beta, r_0, \rho_0)$ and obtain the marginalized distribution $\mathcal{L}_\gamma(\gamma | \widetilde{\Sigma}_{obs})$. This process allows for a fair decoupling of our state of knowledge of γ from $(\alpha, \beta, r_0, \rho_0)$ without, for example, ignoring the data in the outer regions of the dSph in an ad hoc manner.

Care must be taken when choosing the ranges of parameter values to ensure that a fair sampling of the parameter space is made into all regions that can contribute significantly to the marginalized distribution. The reason for this is the edges of the parameter distribution impose the trivial prior that the parameters of candidate models must lie within the range of parameters chosen. For the purposes of this study we have found that a largely unbiased range of parameters is: $\alpha \in [0.5, 12]$; $\beta \in [2.5, 17]$; $\gamma \in [-13, 3]$; $r_0 \in \mathbb{R}$; and $\rho_0 \in \mathbb{R}$.

In practice $\mathcal{L}$ is an extremely rapidly decreasing function of (r_0, ρ_0) about its maximum, and so for computational purposes the marginalization over these variables has been approximated by a maximization. We find that maximization over (r_0, ρ_0) yields indistinguishable results to marginalization provided that the characteristic turnover radius, r_0, is not permitted to be unreasonably large (ie. larger than the edge of the galaxy). Marginalization over (α, β) proceeds straightforwardly to yield $\mathcal{L}_\gamma$.

The three dSphs we have focused our analysis on, Carina, Draco and Ursa Minor have mean ellipticities, $\epsilon = 1 - b/a$, of 0.33, 0.29, and 0.56 respectively and the available surface density data is averaged over elliptical bins. To make a conservative analysis in spherical theory we made a direct comparison of circularly rebinned surface density data with the theoretical profiles, using the properly reweighted error distribution. In general, we find that even the rough approximation of comparing the elliptically binned density profile with spherical theory gives similar limits on γ as the gross dependence of $\mathcal{L}$ on the shape parameters is insensitive to the exact data binning scheme. This is expected as spherically smoothing an ellipsoidal distribution has the effect of smoothing the radial distribution over scales of $\sim \epsilon a$ without significantly altering the gross radial dependence – especially in the nearly flat inner regions important for this analysis.

3 DISCUSSION

The marginalized likelihood distribution $\mathcal{L}_\gamma$ for the three dSphs in question is shown in Figure 1. Although the peak of the distribution fluctuates slightly from galaxy to galaxy the upper bound on γ is consistent between all three.

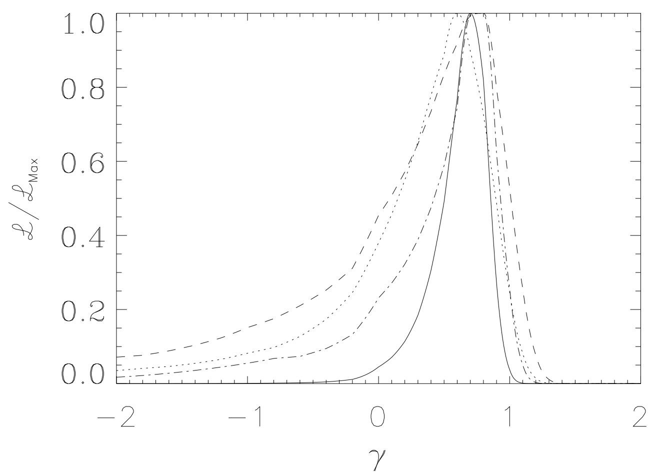

Figure 1. The likelihood distribution of γ, the exponent of the inner power law, for Carina (dot-dashed), Draco (dotted), Ursa Minor (dashed), and for the three galaxies taken together as an ensemble (solid).

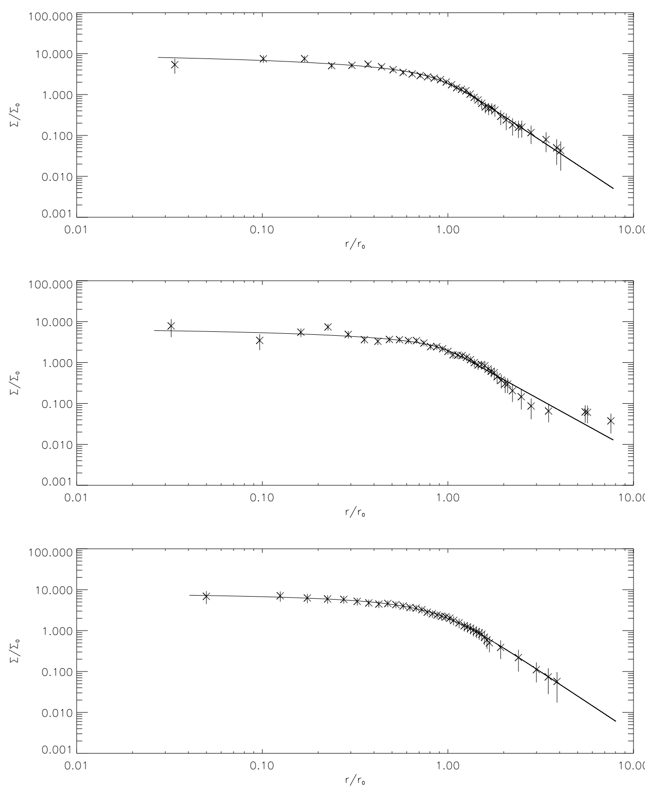

Figure 2. Shown are the maximum likelihood models $(\alpha, \beta, \gamma) = (5.5, 4, 0.7)$ for Carina (Top), $(\alpha, \beta, \gamma) = (9.5, 3.5, 0.6)$ for Draco (Middle), and $(\alpha, \beta, \gamma) = (3.5, 4.0, 0.5)$ for Ursa Minor (Bottom). Models with less sharp turnovers of $\alpha \approx 1 - 2$ can also be found that yield reasonable fits.

While the distributions in γ have tails stretching well into the negative region of parameter space this is not unexpected. Negative values of γ are simply models that have a peak in the density at a scale of $\sim r_0$, a plausible result given the adopted uncertainties. Projection effects coupled with the freedom to renormalize and rescale the density profile have the result of extending the tail relatively far into the negative γ region. Despite the ambiguity between models in the negative γ regime, it is clear from the Figure 1. that models with $\gamma > 1.21$ are strongly excluded by this analysis.

The best-fitting density profiles are shown in Figure 2. Note that while the profiles are nearly always good fits for the inner and central regions there sometimes appears to be an excess of density for the outer few data points – related to the scheme adopted for background subtraction of stars (Irwin & Hatzidimitriou 1995). The results with respect to the parameter γ do not change if these points are included or excluded from the analysis. We have adopted the strategy of including all available data.

Table 1 summarises the limits this analysis places on γ and the other shape parameters for each galaxy on its own, and the galaxies considered together as an ensemble. It also presents the limits with a tophat prior of $1 \leqslant \alpha \leqslant 2$ imposed on the likelihoods – the range of parameter space that the NFW and recent N-body CDM profiles are contained in. The effect of this prior is to shift the peak of the distribution and the upper bound on γ to even *lower* values than when the full range of α is used.

As measurements of the radial profile of the velocity dispersion in these galaxies increase in sensitivity a joint analysis fitting both the stellar dispersion and stellar density profiles predicted by a given DM profile should provide a much better test of the CDM and similar paradigms.

Table 1. Summary of shape parameter limits.

Uniform Priors				
Galaxy	$\gamma(99\%)$*	γ_{peak}	β (2σ)	α (2σ)
Carina	<1.09	0.80	$4.0^{+2.3}_{-0.6}$	<5.2
Draco	<1.12	0.60	$3.5^{+1.9}_{-0.3}$	<7.3
Ursa Minor	<1.21	0.80	$4.0^{+4.5}_{-0.8}$	<2.7
Ensemble	<0.98	0.70	$4.0^{+0.8}_{-0.5}$	-
$1 \leqslant \alpha \leqslant 2$ Tophat Prior				
Carina	<0.76	0.00	$5.0^{+4.9}_{-1.1}$	-
Draco	<0.76	0.00	$5.0^{+5.0}_{-1.4}$	-
Ursa Minor	<0.95	0.00	$5.0^{+6.8}_{-1.5}$	-
Ensemble	<0.52	0.00	$5.0^{+2.8}_{-0.8}$	-

*These values are based on the one sided Gaussian positive deviation from the peak. Calculating with the full two sided distribution yields an even stronger upper confidence bound on γ.

4 CONCLUSION

We have completed a Bayesian likelihood analysis of the stellar density profile shapes of the Galactic dwarf spheroidal galaxies Draco, Ursa Minor, and Carina. With uniform prior assumptions for the parameters $(\alpha, \beta, \gamma, r_0, \rho_0)$ we conclude that, for each galaxy, $\gamma < 1.21$ at greater than 99% confidence.

As discussed in the introduction, except in the special situation where the bulk of the stars formed before the dark matter halo, γ_{DM} would not be expected to be the same as γ_{stars}. Nevertheless the comparison we made quantifies the

the discrepancy between the observed inner stellar profile of dSphs and simulated dark matter haloes, a discrepancy which CDM theory or one of its cousins must ultimately explain.

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